Warm-up

Solve the differential equation $\frac{dy}{dx} = 6x^2 y$

with the condition $y(1) = 3$.

$dy = 6x^2 y\, dx$

$\int \frac{dy}{y} = \int 6x^2\, dx$

$\ln|y| = 2x^3 + C$

$\ln|3| = 2(1)^3 + C$

$\ln 3 = 2 + C$

$(\ln 3) - 2 = C$

$e^{\ln|y|} = e^{2x^3 + \ln 3 - 2}$

$|y| = e^{2x^3 + \ln 3 - 2}$

$y = e^{2x^3 + \ln 3 - 2}$ OR $\cancel{y = -e^{2x^3 + \ln 3 - 2}}$

$y = \frac{e^{2x^3} \cdot e^{\ln 3}}{e^2} = \frac{3e^{2x^3}}{e^2}$

$x^4 \cdot x^5 = x^9$

$x^m \cdot x^n = x^{m+n}$

$\frac{x^m}{x^n} = x^{m-n}$

Warm-up #2

Evaluate.

No Calculator #1-3

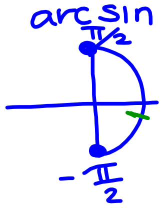
Ex 1. $\arcsin\left(-\frac{1}{2}\right)$ $\quad -\frac{\pi}{6}$ $\quad \cancel{\frac{7\pi}{6}}\ \cancel{\frac{11\pi}{6}}$

Ex 2. $\arccos(0)$ $\quad \frac{\pi}{2}$, $\cancel{\frac{3\pi}{2}}$

Ex 3. $\tan^{-1}(\sqrt{3})$ $\quad \frac{\pi}{3}$ and $\cancel{\frac{2\pi}{3}}$

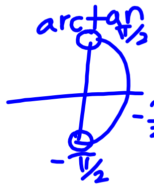
Ex 4. $\arcsin(0.3)$ $\quad .3046$

arcsin x



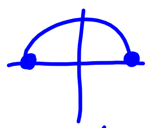
$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

arctan



$-\frac{\pi}{2} < y < \frac{\pi}{2}$

arccos x



$0 \le y \le \pi$

[5.6] Inverse Trigonometric Functions: Differentiation

Definitions of Inverse Trig Functions:

	Domain	Range
$y = \arcsin x$ iff $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$ iff $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$ iff $\tan y = x$	$-\infty \le x \le \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \text{arc}\cot x$ iff $\cot y = x$	$-\infty \le x \le \infty$	$0 < y < \pi$
$y = \text{arc}\sec x$ iff $\sec y = x$	$\lvert x \rvert \ge 1$	$0 \le y \le \pi, \quad y \ne \frac{\pi}{2}$
$y = \text{arc}\csc x$ iff $\csc y = x$	$\lvert x \rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, \quad y \ne 0$

Let's Explore

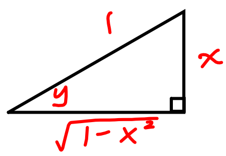
y = sin^{-1}(x)

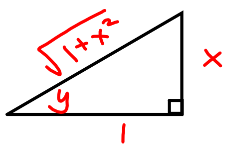
$\sin y = x$

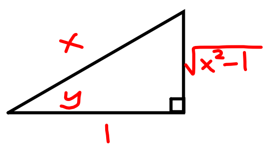
$\cos y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$







y = tan^{-1}(x)

$\tan y = x$

$\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

y = sec^{-1}(x)

$\sec y = x$

$\sec y \tan y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$

$\frac{dy}{dx} = \frac{1}{\lvert x \rvert \sqrt{x^2-1}}$

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\operatorname{arc\,sec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\operatorname{arc\,csc} u] = -\frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\operatorname{arc\,cot} u] = -\frac{u'}{1+u^2}$$

Differentiate.

Ex 1. $y = \arcsin(2x)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

Ex 2. $y = \sec^{-1}\left(e^{x^2}\right)$

$$\frac{dy}{dx} = \frac{1}{|e^{x^2}|\sqrt{(e^{x^2})^2-1}} \cdot e^{x^2} \cdot 2x$$

OR

$$\frac{2x\,\cancel{e^{x^2}}}{\cancel{e^{x^2}}\sqrt{e^{2x^2}-1}} \quad \text{OR} \quad \frac{2x}{\sqrt{e^{2x^2}-1}}$$

Ex 3. $y = \arctan\left(\sqrt{3x}\right)$

$$\frac{dy}{dx} = \frac{1}{\left(\sqrt{3x}\right)^2 + 1} \cdot \frac{1}{2}(3x)^{-1/2} \cdot 3$$

Ex 4. $y = \arcsin(x) + x\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + (x)\left(\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right) + \sqrt{1-x^2}$$

$$\text{OR} \quad \frac{1}{\sqrt{1-x^2}} - \frac{x^2}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{1} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$\text{OR} \quad \frac{1-x^2+1-x^2}{\sqrt{1-x^2}} = \frac{2-2x^2}{\sqrt{1-x^2}}$$